Self-similar demostration 1. The Einstein tensor and the Kretschmann scalar are calculated for the spherically symmetric self-similar metric in curvature coordinates.

> restart:
> interface(labelling=false):
> grtw();

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Latest version available from: http://grtensor.phy.queensu.ca/
e:/Grtii(6)/Metrics

> qload(ss1):

Default spacetime = ss1
For the ss1 spacetime:
Coordinates
x(up)
\( \mathbf{x}^a = [r, \theta, \phi, t] \)
Line element
\[
\begin{align*}
\left( \begin{array}{cccc}
\lambda & 0 & 0 & \frac{\lambda}{r} \\
0 & r^2 & 0 & 0 \\
0 & 0 & r^2 \sin^2 \theta & 0 \\
\frac{\lambda}{r} & 0 & 0 & \lambda \frac{r^2}{r^2} \\
\end{array} \right)
\end{align*}
\]

First we verify that the vector \( \xi^a = c(r,0,0,t) \) is indeed homothetic.
> #grdefine(`xi{^a}`,{},[c*r,0,0,c*t]);
> grdef(`xi{^a}:=[c*r,0,0,c*t]`);

Components assigned for metric: ss1
Created definition for xi(up)

> grcalc(KillingTest[xi]);

Created a definition for xi(up,cdn)
Created a definition for xi(dn)
Created a definition for xi(dn,cdn)

CPU Time = .311

> grdisplay(_);

For the ss1 spacetime:

Killing Test Result

Killing Test[xi] = ( a homothetic/conformal Killing vector with conformal factor = c )

We now calculate the mixed Einstein tensor and the Kretschmann scalar.
> grcalc(G(dn,up),RiemSq);

Created definition for G(dn,up)
Created definition for R(dn,dn,up,up)

CPU Time = .360
The following substitutes $\xi=r/t$.

$$> \text{grmap}(\_,\text{subs}, t=r/\xi, \text{`x'}):$$

Applying routine subs to $G(dn,up)$
Applying routine subs to RiemSq

We now convert to $\xi$ derivatives.

$$> \text{grmap}(\_,\text{convert}, \text{`x'}, \text{diff}, \xi):$$

Applying routine convert to $G(dn,up)$
Applying routine convert to RiemSq

The quantities are simplified and factored.

$$> \text{gralter}(\_,1,7);$$

Component simplification of a GRTensorII object:
Applying routine simplify to object $G(dn,up)$
Applying routine simplify to object RiemSq
Applying routine factor to object $G(dn,up)$
Applying routine factor to object RiemSq

\[ CPU \ Time \ = \ .291 \]

In order to write the $\xi$ derivatives as subscripts we use the autoAlias feature in the grtools package.

$$> \text{grmap}(\_,\text{autoAlias}, \text{`x'}):$$

Applying routine autoAlias to $G(dn,up)$

We now display the results.

$$> \text{grdisplay}(\_);$$

For the ss1 spacetime:

\[
G(dn,up)
\]

\[
G(dn, up )
\]

\[
G_r^r = -\frac{\left( e^{-\lambda(\xi)} \lambda^2 \xi^2 \right)}{r^2}
\]

\[
G_t^r = -\frac{\left( e^{-\lambda(\xi)} \lambda^2 \xi^2 \right)}{r^2}
\]

\[
G_\theta^\theta = \frac{1}{4} \left( (-2 \lambda e^v + 2 \lambda e^v + \lambda^2 v^2 e^v + \lambda e^v \xi^2 - 2 \lambda e^v \xi^3 - 4 \lambda e^v \xi^3 - \lambda^2 \xi^3 \right) \frac{\left( e^{-\lambda(\xi)} \right)}{r^2}
\]

\[
G_\phi^\phi = \frac{1}{4} \left( (-2 \lambda e^v + 2 \lambda e^v + \lambda^2 v^2 e^v + \lambda e^v \xi^2 - 2 \lambda e^v \xi^3 - 4 \lambda e^v \xi^3 - \lambda^2 \xi^3 \right) \frac{\left( e^{-\lambda(\xi)} \right)}{r^2}
\]

\[
G_r^t = \frac{\left( e^{-\lambda(\xi)} \lambda^2 \xi^2 \right)}{r^2}
\]

\[
G_t^t = -\frac{\left( \lambda \xi^2 + e^\lambda x - 1 \right) e^{-\lambda(\xi)}}{r^2}
\]
Full Contraction of Riemann

\[ K = \frac{1}{4} \left( (4 \lambda_{\xi} e^{(v+\lambda(\xi))} \lambda_{\xi} \nu_{\xi} \xi^{6} + 4 \nu_{\xi,\xi} e^{(v+\lambda(\xi))} \lambda_{\xi} \nu_{\xi} \xi^{6} - 32 e^{(2 v+\lambda(\xi))} \right) \]

Note that Kretschmann is of the form $f(\xi)/r^{n}$. This holds for all the algebraic invariants.